**HEAT TRANSPORT AND DIFFUSION**

Due date: 3/22/2016

Partial differential equations come in three general forms: hyporbolic (i.e. wave equation, $\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$), parabolic (i.e. diffusion equation, $\nabla^2 u - \frac{1}{D}\frac{\partial u}{\partial t} = 0$) and elliptic (i.e. Poisson equation, $\nabla^2 u = \rho(x, y, z)$). More importantly, we distinguish *initial value* problems and *boundary value* problems. The main issue with numerical integration of initial value problems is stability, while the main issue with boundary value problems is efficiency. Today we will explore an initial value problem: a rod placed between two large, cool reservoirs with $T = 0$. A segment of it (say, between 0.3 and 0.5 of its length) is heated to $T = 1$. We are interested in time propagation of cooling. This is described with a 1-dimensional diffusion equation:

$$\frac{\partial u}{\partial t} - D\nabla^2 u \equiv \frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} = 0,$$

where $D$ is the diffusion constant.

1. Explore stability criteria for several different finite difference schemes, i.e. FTCS, Lax, downwind, upwind, leap-frog, Crank-Nicolson, Lax-Wendroff, etc. Which ones are unconditionally unstable, unconditionally stable, or conditionally stable?

2. Add heating of the rod at a distinct segment, i.e. between 0.6 and 0.8 of the rod length. This is done by making the equation non-homogenous:

$$\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} = \frac{q(x)}{\rho c},$$

   where $q(x)$ are heat sources, $\rho$ is the density of the rod, and $c$ is its thermal capacity. How does diffusion change then?

3. Perform a similar analysis for a string with a triangular initial condition. Observe the propagation of the triangular wave in time.

4. *Extra credit:* Evaluate stability criteria analytically, using the Von Neumann criterion.