# POISEUILLE'S LAW

Due date: 3/31/2016

Continuing our surface scratching of partial differential equations, this week we deal with boundary conditions. There are rarely ever stability problems associated with boundary value problems; instead, your main concern here is efficiency. The problem we tackle today is continuous fluid flow through pipes with a non-circular cross-section. For incompressible fluids in general, this problem is described by the Navier-Stokes equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \eta \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g},$$

where $\mathbf{u}$ is a velocity field, $\eta$ is kinematic viscosity, $w$ is external agent work per unit mass, and $\mathbf{g}$ is any external force per unit mass. This equation simplifies significantly for incompressible newtonian fluids that exhibit laminar flow through pipes with a constant cross-section:

$$\nabla^2 \mathbf{u} = -\frac{\Delta p}{\eta L},$$

where $\Delta p$ is the pressure difference across the length of the pipe $L$. This is the Poisson equation, and the subject of this week's assignment.

1. This assignment has a theoretical part. In class we derived the Poiseuille law for a circular aperture. Using help from the literature (unless you feel like trying out your analytical PDE solving skills), do the same for the elliptical, semi-circular, rectangular and equilaterally triangular aperture. You can find a potentially helpful paper on this topic at the course webpage.

2. Build a PDE model for the above apertures and iterate it until you obtain a stationary solution. Use density plot, heatmap, and/or surface plot to visualise the flow velocity field.

3. Determine the Poiseuille coefficient for each of these apertures and compare it with the analytical values.

4. Make your own aperture – such that a "simple" analytical solution does not exist – and solve it numerically. Let your imagination go wild.