POPULATION MODELS

Due date: 2/15/2018

We have seen in our previous exercise that Runge-Kutta methods were great for solving perfectly integrable differential equations. In the case of non-linear equations this is no longer guaranteed and hybrid methods, such as the Livermore Solver (LSODE; `odeint` in GSL and scipy), prove to be better suited for this problem.

1. **Logistic equation.** After the initial hype (and failure) of the basic (exponential) population growth model, we need to incorporate resource limitations into the model. These are described with the logistic equation (the Verhulst model):

$$\dot{P} = kP\left(1 - \frac{P}{N}\right),$$

where P is population size, k is the growth rate, and N is the size constraint. Using the table of US population from the course webpage, compare the fit of the logistic model to the fit of the exponential growth model. What are suitable values of k and N?

2. **Predator–prey.** Determine the number of independent parameters for the Lotka-Volterra model, draw and carefully analyze the phase diagram. Determine the population equilibrium (the point in the phase diagram where the population levels of foxes and rabbits are not changing) and points of stability. What happens if you add wolves into the system?

3. **The epidemic equation.** Divide the population into three groups: (1) healthy, (2) sick, and (3) immune. The desease spreads by contact between the sick and the healthy with some constant probability. The sick become immune (either healthy or dead) by another constant probability. Form the model and determine the minimum number of independent parameters to describe it. During the epidemic, we are interested in: the time of peak in the number of sick people; the overall number of sick people; and the maximum number of sick people at one time. Determine these quantities for some realistic values of parameter(s). How does immunotherapy (shots) change these results?