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Due date: 2/22/2018

In our previous exercise we used differential equations to predict the evolution of an epidemic. In this exercise we will do the same, only this time we will use a stochastic approach to modeling.

1. The transition between states $S_n \to S_{n+1}$ can be described by the Markov matrix (a.k.a. the *transition* or *stochastic* matrix) $\mathcal{M}$. Assemble a Markov matrix for the gambler problem discussed in class and predict the long-term outcome for the simplified version of roulette.

2. Build a city with population density $\sigma = N/A$ and uniform areal distribution. Determine the time it takes for an epidemic to spread throughout the population. Measure the speed of disease propagation in terms of stochastic steps. First evaluate limiting cases ($\sigma \to 0$, $\sigma \to 1$), then do a general case ($N_{\rm steps}(\sigma)$). Compare that to the results from the previous example.

3. You can do a similar exercise for the rabbits and foxes. This time around there is no such thing as 10^{-5} rabbits where the population can still recover. Compare the "orbital" time, and show that population death is an inevitable consequence predicted by discrete population models.

4. For more realistic examples, the continuous (ODE-based) model becomes overly complicated, but the stochastic approach remains straightforward. Test it on different population distributions, include recovery, immunization, recurring diseases, diseases in rabbits/foxes, meteorite impacts, etc.