THE RESTRICTED AND NON-RESTRICTED 3-BODY PROBLEM

Due date: 1/30/2020, 9am

Continuing with our exploration of ordinary differential equations (and celestial mechanics), here we address the restricted and non-restricted three-body problems. The term "restricted" pertains to a *massless* particle moving in the gravitational field of two massive objects that orbit their mutual center of mass.

a) Plot $\psi(r)$ and $\mathbf{F}(\mathbf{r})$ in 1D, 2D and 3D for a restricted 3-body problem. Plot the gravitational potential/force in 1D for comparison.

b) Determine the location of Lagrangian points and study their stability. In particular, determine the value of $q = M_2/M_1$ for which L_4 and L_5 become unstable.

c) Using the Runge-Kutta and/or Bulirsch-Stoer integrators, study the orbits of Trojan and Greek asteroids. How sensitive are the orbits to initial conditions? Can you find initial conditions that result in periodic orbits?

d) Plot the shapes of stars for different values of the equipotential.

e) How do the orbits change if M_3 is no longer vanishingly small?

Do not forget that you are solving this in a rotating coordinate system. Hence you need to be careful about the initial conditions and about the Coriolis force.

A quick reminder from astrodynamics: Lagrange points are given by:

$$L_1 = a\left[1-\left(\frac{\eta}{3}\right)^{1/3}\right], \quad L_2 = a\left[1+\left(\frac{\eta}{3}\right)^{1/3}\right], \quad L_3 = -a\left[1-\left(\frac{5\eta}{12}\right)^{1/3}\right], \tag{1}$$

$$L_4 = a\left(\frac{1}{2}\frac{M_1-M_2}{M_1+M_2}, \frac{\sqrt{3}}{2}\right), \quad L_5 = a\left(\frac{1}{2}\frac{M_1-M_2}{M_1+M_2}, -\frac{\sqrt{3}}{2}\right), \tag{2}$$

where a is the semi-major axis and $\eta = M_2/(M_1+M_2)$.