DYNAMICAL CHAOS

Due date: 3/20/2024, 9am

Dynamical systems are systems where a function describes the time dependence of a point in a geometrical space. The system is described by a *state* at any given time t that is described by the parameter vector $\mathbf{p}$. Dynamical systems evolve by the *evolution rule*, which is a function that prescribes the propagation of the current state to a future state. Evolution rules can be either deterministic or stochastic. The time propagation of a non-linear system can depend quite strongly on the initial conditions, leading to an essentially unpredictable state of the system. This is known as dynamical chaos.

1. Plot the logistic map for the Feigenbaum set and examine it carefully. Determine the bifurcation points accurately and determine the Feigenbaum constant. Study the stability of attractor(s) for several values of the growth parameter r. Show by graphing that the Feigenbaum diagram is self-similar.

2. Plot the Mandelbrot set for different values of c. Color-code by the number of iterations required to cross a given threshold. Show by graphing (hint hint, animations are awesome!) that the Mandelbrot set is a fractal.

3. Determine the cycle period of Arnold's cat map for a chosen dimension N. Can you say anything about the minimum or maximum values of the cycle period as a function of N?

4. For the standard (Taylor-Greene-Chirikov) map plot the phase diagram and determine the critical value(s) for the kick parameter γ. Find fractal islands and determine the value of γ at which they dissapear. How many attractors does the standard map have?

5. Include attenuation to the standard map and find attractors as function of the attenuation coefficient. This is known as the Zaslavskii map.