EXAMPLES OF NUMERICAL PROBLEMS INVOLVING THE H-R DIAGRAM

The Hertzsprung-Russell diagram in a very fancy form is included on the last page; use it when necessary.

1. Let us compare a star with $2\,M_\odot$ to the star with $0.5\,M_\odot$.
   a) Compute the luminosities of both stars. Express your answers in solar luminosity.
   b) Using the H-R diagram, estimate the surface temperature of both stars. Express the answers in solar temperature.
   c) Using the luminosity and the temperature, compute the radii of both stars. Express your answers in solar radii.
2. Now let us focus on magnitudes.
   a) How much brighter is a star of magnitude 0 from the star of magnitude 1? 2? 3? 4? 5?
   b) if the absolute magnitude of a star is larger than the absolute magnitude of the Sun by 5, what is the luminosity of that star, expressed in solar luminosity?
3. Next let us focus on the distance modulus.
   a) Express the distance to the Sun in parsecs.
   b) What is the absolute magnitude of the Sun if its apparent magnitude is -27?
   c) If we observe a Sun-like star with the apparent magnitude of 12, how far away is it?
4. More fun with the distance modulus.
   a) Two stars have the same apparent magnitude. One is 3 times farther than the other. What is their luminosity ratio?
   b) Two stars are at the same distance. One is 3 times brighter than the other. What is their absolute magnitude difference?
   c) Two stars have the same absolute magnitude. One is 3 times farther than the other. What is their apparent magnitude difference?

5. Finally, let us put everything together. Let us again consider our two stars, one with $2\,M_\odot$ and the other with $0.5\,M_\odot$.

   a) Compute again the luminosities of the two stars. Do it again so that you remind yourself that luminosity depends on temperature *and* size.

   b) Compute the difference in absolute magnitudes of both stars.

   c) Compute absolute magnitudes of both stars. You will have to know the absolute magnitude of the Sun for this.

   d) Compute the distance ratio if both stars appear equally bright.

6. One last problem. From the H-R diagram, read off the approximate luminosity of (a) a red supergiant, (b) a red giant, and (c) a red dwarf. Make sure that your reading is done at approximately the same temperature.

   a) Determine the masses of all three stars; express them in solar mass.

   b) Determine the radii of all three stars; express them in solar radii.

   c) If all three stars were at a distance of 10 parsecs, what would their apparent magnitudes be?

   d) If all three stars were at a distance of our Sun, how much brighter/ fainter than the Sun would they be?

HOMEWORK KEY:

(1) $M = 2M_\odot$ $\qquad$ $M = 0.5\,M_\odot$

(a) $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} = \underline{\underline{11.3}}$ $\qquad$ $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} = \underline{\underline{0.09}}$

(b) $T = 9000K$ $\qquad$ $T = 4000\,K$

$\frac{T}{T_\odot} = \underline{\underline{1.5}}$ $\qquad$ $\frac{T}{T_\odot} = \underline{\underline{0.67}}$

(c) $\left(\frac{L}{L_\odot}\right) = \left(\frac{T}{T_\odot}\right)^4 \left(\frac{R}{R_\odot}\right)^2$

$\left(\frac{R}{R_\odot}\right) = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T}{T_\odot}\right)^{-2}$

$\frac{R}{R_\odot} = \underline{\underline{1.5}}$ $\qquad$ $\frac{R}{R_\odot} = \underline{\underline{0.67}}$

(2) (a) $m_1 - m_2 = -\frac{5}{2} \log \frac{f_1}{f_2}$

$\Rightarrow \cancel{\log} \frac{f_1}{f_2} = 10^{-\frac{2}{5}(m_1 - m_2)}$

$\frac{f_0}{f_1} = 10^{\frac{2}{5}} = \underline{\underline{2.512}}$ $\qquad$ $\frac{f_0}{f_2} = 10^{\frac{4}{5}} = \underline{\underline{6.31}}$

$\frac{f_0}{f_3} = 10^{\frac{6}{5}} = \underline{\underline{15.85}}$ $\qquad$ $\frac{f_0}{f_4} = 10^{\frac{8}{5}} = \underline{\underline{39.85}}$

$\frac{f_0}{f_5} = 10^{2} = \underline{\underline{100}}$

(b) $M - M_\odot = 5$

$$M - M_\odot = -\frac{5}{2} \log \frac{L}{L_\odot}$$

$$\Rightarrow \frac{L}{L_\odot} = 10^{-\frac{2}{5}(M - M_\odot)} = 10^{-\frac{2}{5}5} = 10^{-2} = \underline{\underline{0.01}}$$

(3) (a)

$$\tan(1'') = \tan(1/3600^\circ) = \frac{1\,pc}{D}$$

$$\Rightarrow D = \frac{1\,pc}{\tan(1/3600^\circ)} = 206{,}265\,au$$

since by def. $D = 1\,pc$: $1\,au = \frac{1}{D} = \underline{\underline{4.85 \times 10^{-6}\,pc}}$

(b) $m_\odot - M_\odot = 5 \log D - 5$

$$\Rightarrow M_\odot = m_\odot - 5 \log D + 5 \text{ , } D \text{ in pc!}$$

$M_\odot = \underline{\underline{4.57}}$ (really it is closer to 4.85)

(c) $m = 12$, $D = ?$

$$m - M = 5 \log D - 5$$

$$\Rightarrow D = 10^{\frac{1}{5}(m - M + 5)} = 10^{\frac{1}{5}(12 - 4.57 + 5)}$$

$$= \underline{\underline{306\,pc.}}$$

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(a) $m_1 = m_2$ $\qquad$ $m_1 - m_2 = -\frac{5}{2}\log\frac{f_1}{f_2}$

$D_1 = 3D_2$ $\qquad$ recall that $L = f \times 4\pi D^2$

$$m_1 - m_2 = -\frac{5}{2}\log\frac{L_1}{\cancel{4\pi} D_1^2}\frac{\cancel{4\pi} D_2^2}{L_2} = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2$$

$$0 = \log\left(\frac{L_1}{L_2}\right)\left(\frac{1}{3}\right)^2 \Rightarrow \left(\frac{L_1}{L_2}\right)\left(\frac{1}{3}\right)^2 = 1$$

$$\underline{\underline{\left(\frac{L_1}{L_2}\right) = 9}}$$

(b) $D_1 = D_2$

$M_1 - M_2 = ?$

$\frac{f_2}{f_1} = 3$

SINCE $D_1 = D_2$, IT DOES NOT MATTER WHETHER WE THINK OF APPARENT OR ABSOLUTE MAGS, THE DIFFERENCE WILL BE THE SAME.

$$m_1 - m_2 = -\frac{5}{2}\log\frac{f_1}{f_2} = -\frac{5}{2}\log\frac{1}{3} = \underline{\underline{1.2}}$$

(c) $M_1 = M_2$

$D_1 = 3D_2$

SINCE $M_1 = M_2$, IT FOLLOWS THAT:

$L_1 = L_2$

FROM THE INVERSE SQUARE LAW (IDENTICAL TO 4a):

$$m_1 - m_2 = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2 = -\frac{5}{2}\log\left(\frac{1}{3}\right)^2$$

$$= \underline{\underline{2.38}}$$

⑤ (a) $L_{2M_\odot} = 11.3\,L_\odot \qquad L_{0.5M_\odot} = 0.09\,L_\odot$

(b) $M_1 - M_2 = -\frac{5}{2}\log\frac{L_1}{L_2} = -\frac{5}{2}\log\frac{11.3\,\cancel{L_\odot}}{0.09\,\cancel{L_\odot}}$

$$= \underline{\underline{-5.25}}$$

(c) $M_1 - M_\odot = -\frac{5}{2}\log\frac{L_1}{L_\odot} \Rightarrow M_1 = M_\odot - \frac{5}{2}\log\frac{L_1}{L_\odot}$

$\underline{\underline{M_1 = 2.22}}$, and equivalently $\underline{\underline{M_2 = 7.46}}$

(d) $m_1 - m_2 = 0$

$m_1 - m_2 = -\frac{5}{2}\log\frac{f_1}{f_2} = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2$

$\frac{L_1}{L_2} = \left(\frac{D_1}{D_2}\right)^2 \Rightarrow \frac{D_1}{D_2} = \left(\frac{L_1}{L_2}\right)^{1/2} = \left(\frac{11.3}{0.09}\right)^{1/2}$

$$= \underline{\underline{11.2}}$$

⑥ $T = 4500\,K$

$L_{SG} = 10^5 L_\odot, \quad L_G = 10^2 L_\odot \quad L_D = 0.1\,L_\odot$

(a) $\frac{M_{SG}}{M_\odot} = \left(\frac{L_{SG}}{L_\odot}\right)^{1/3.5} = \underline{\underline{26.8}}$

$\frac{M_G}{M_\odot} = \left(\frac{L_G}{L_\odot}\right)^{1/3.5} = \underline{\underline{3.72}}$

$\frac{M_D}{M_\odot} = \left(\frac{L_D}{L_\odot}\right)^{1/3.5}$

$= \underline{\underline{0.52}}$

(b) $\frac{L}{L_\odot} = \left(\frac{T}{T_\odot}\right)^4 \left(\frac{R}{R_\odot}\right)^2$

$$\Rightarrow \left(\frac{R}{R_\odot}\right) = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T}{T_\odot}\right)^{-2}$$

$$\left(\frac{R_{SG}}{R_\odot}\right) = \underline{\underline{562}} \qquad \left(\frac{R_G}{R_\odot}\right) = \underline{\underline{17.8}}$$

$$\left(\frac{R_D}{R_\odot}\right) = \underline{\underline{0.56}}$$

(DO YOU SEE NOW WHY THEY'RE CALLED SUPER-GIANTS, GIANTS AND DWARFS?)

(c) APPARENT MAG @ 10 pc <u>IS</u> ABSOLUTE MAGNITUDE:

$$M_{SG} - M_\odot = -\frac{5}{2}\log\frac{L_{SG}}{L_\odot} = \underline{\underline{-12.5}}$$

$$M_G - M_\odot = -\frac{5}{2}\log\frac{L_G}{L_\odot} = \underline{\underline{-5}}$$

$$M_D - M_\odot = -\frac{5}{2}\log\frac{L_D}{L_\odot} = \underline{\underline{+2.5}}$$

(d) $m_{SG} - M_{SG} = 5\log D - 5$

$$\Rightarrow m_{SG} = M_{SG} + 5\log D - 5$$

$$= M_\odot - \frac{5}{2}\log\frac{L_{SG}}{L_\odot} + 5\log D - 5$$

$$= \underline{-39.2}$$

(5)

$$m_G = M_\odot - \frac{5}{2}\log\frac{L_G}{L_\odot} + 5\log D - 5$$

$$= \underline{\underline{-31.7}}$$

$$m_D = M_\odot - \frac{5}{2}\log\frac{L_D}{L_\odot} + 5\log D - 5$$

$$= \underline{\underline{-24.2}}$$

IF YOU SOLVED THIS IN FLUXES, IT'S JUST AN EXTRA STEP:

$$m - m_\odot = -\frac{5}{2}\log\frac{f}{f_\odot} \Rightarrow \frac{f}{f_\odot} = 10^{-\frac{2}{5}(m - m_\odot)}$$

$$\frac{f_{SG}}{f_\odot} = \underline{\underline{75{,}858}}, \quad \frac{f_G}{f_\odot} = \underline{\underline{75.8}}, \quad \frac{f_D}{f_\odot} = \underline{\underline{0.08}}$$