EXAMPLES OF NUMERICAL PROBLEMS INVOLVING THE H-R DIAGRAM

1. Let us compare a star with $2\,M_\odot$ to the star with $0.5\,M_\odot$.

   a) Compute the luminosities of both stars. Express your answers in solar luminosity.

$$\mathcal{M} = 2\mathcal{M}_\odot \quad \Rightarrow \quad \frac{L}{L_\odot} = \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5} = 11.3$$

$$\mathcal{M} = 0.5\mathcal{M}_\odot \quad \Rightarrow \quad \frac{L}{L_\odot} = \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5} = 0.09$$

   b) Using the H-R diagram, estimate the surface temperature of both stars. Express the answers in solar temperature.

$$T_1 \approx 9000\,\mathrm{K} \quad \Rightarrow \quad \left(\frac{T_1}{T_\odot}\right) \approx 1.6$$

$$T_2 \approx 4000\,\mathrm{K} \quad \Rightarrow \quad \left(\frac{T_2}{T_\odot}\right) \approx 0.7$$

   c) Using the luminosity and the temperature, compute the radii of both stars. Express your answers in solar radii.

$$\left(\frac{R_1}{R_\odot}\right) = \left(\frac{L_1}{L_\odot}\right)^{1/2} \left(\frac{T_1}{T_\odot}\right)^{-2} = 1.31$$

$$\left(\frac{R_2}{R_\odot}\right) = \left(\frac{L_2}{L_\odot}\right)^{1/2} \left(\frac{T_2}{T_\odot}\right)^{-2} = 0.61$$

2. Now let us focus on magnitudes.

   a) How much brighter is a star of magnitude 0 from the star of magnitude 1? 2? 3? 4? 5?

$$m_1 = m_2 = -\frac{5}{2}\log\frac{f_1}{f_2} \quad \Rightarrow \quad \frac{f_1}{f_2} = 10^{-\frac{2}{5}(m_1-m_2)}$$

$$\frac{f_0}{f_1} = 10^{2/5} = 2.512, \quad \frac{f_0}{f_2} = 10^{4/5} = 6.31, \quad \frac{f_0}{f_3} = 10^{6/5} = 15.85,$$

$$\frac{f_0}{f_4} = 10^{8/5} = 19.05, \quad \frac{f_0}{f_5} = 10^2 = 100$$

b) if the absolute magnitude of a star is larger than the absolute magnitude of the Sun by 5, what is the luminosity of that star, expressed in solar luminosity?

$$M - M_\odot = 5 = -\frac{5}{2}\log\left(\frac{L}{L_\odot}\right)$$

$$\Rightarrow \quad \left(\frac{L}{L_\odot}\right) = 10^{-\frac{2}{5}(M-M_\odot)} = 10^{-2} = 0.01$$

3. Next let us focus on the distance modulus.

a) Express the distance to the Sun in parsecs.

$$\tan 1'' = \frac{1\,\text{pc}}{D} \quad \Rightarrow \quad D = \frac{1\,\text{pc}}{\tan 1''} = 4.85 \times 10^{-6}\,\text{pc}$$

b) What is the absolute magnitude of the Sun if its apparent magnitude is -26.74?

$$m_\odot - M_\odot = 5\log D - 5 \quad \Rightarrow \quad M_\odot = m_\odot - 5\log D + 5 = 4.83$$

c) If we observe a Sun-like star with the apparent magnitude of 12, how far away is it?

$$m - M = 5\log D - 5 \quad \Rightarrow \quad D = 10^{\frac{1}{5}(m-M+5)} = 10^{\frac{1}{5}(12-4.83+5)} = 272\,\text{pc}$$

4. More fun with the distance modulus.

a) Two stars have the same apparent magnitude. One is 3 times farther than the other. What is their luminosity ratio?

$$m_1 = m_2; D_1 = 3D_2$$

$$m_1 - m_2 = -\frac{5}{2}\log\frac{f_1}{f_2} = -\frac{5}{2}\log\frac{L_1}{4\pi D_1^2}\frac{4\pi D_2^2}{L2} = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2$$

$$m_1 - m_2 = 0 = log\left(\frac{L_1}{L_2}\right)\left(\frac{1}{3}\right)^2 \quad \Rightarrow \quad \left(\frac{L_1}{L_2}\right) = 9$$

b) Two stars are at the same distance. One is 3 times brighter than the other. What is their absolute magnitude difference?

$$D_1 = D_2; \frac{f_2}{f_1} = 3$$

$$m_1 - m_2 = -\frac{5}{2}\log\frac{f_1}{f_2} = -\frac{5}{2}\log\frac{1}{3} = 1.2$$

c) Two stars have the same absolute magnitude. One is 3 times farther than the other. What is their apparent magnitude difference?

$$M_1 = M_2; D_1 = 3D_2$$

$$m_1 - m_2 = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2 = -\frac{5}{2}\log\left(\frac{1}{3}\right)^2 = 2.38$$

5. Finally, let us put everything together. Let us again consider our two stars, one with $2\,M_\odot$ and the other with $0.5\,M_\odot$.

   a) Compute again the luminosities of the two stars. Do it again so that you remind yourself that luminosity depends on temperature *and* size.

$$\mathcal{M} = 2\mathcal{M}_\odot \quad \Rightarrow \quad \frac{L}{L_\odot} = \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5} = 11.3$$

$$\mathcal{M} = 0.5\mathcal{M}_\odot \quad \Rightarrow \quad \frac{L}{L_\odot} = \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5} = 0.09$$

   b) Compute the difference in absolute magnitudes of both stars.

$$M_1 - M_2 = -\frac{5}{2}\log\frac{L_1}{L_2} = -\frac{5}{2}\log\frac{11.3}{0.09} = -5.25$$

   c) Compute absolute magnitudes of both stars. You will have to know the absolute magnitude of the Sun for this.

$$M_1 - M_\odot = -\frac{5}{2}\log\left(\frac{L_1}{L_\odot}\right) \quad \Rightarrow \quad M_1 = M_\odot - \frac{5}{2}\log\left(\frac{L_1}{L_\odot}\right) = 2.22$$

$$M_2 - M_\odot = -\frac{5}{2}\log\left(\frac{L_2}{L_\odot}\right) \quad \Rightarrow \quad M_2 = M_\odot - \frac{5}{2}\log\left(\frac{L_2}{L_\odot}\right) = 7.46$$

   d) Compute the distance ratio if both stars appear equally bright.

$$m_1 = m_2 \quad \Rightarrow -\frac{5}{2}\log\frac{f_1}{f_2} = -\frac{5}{2}\log\left(\frac{L_1}{L_2}\right)\left(\frac{D_2}{D_1}\right)^2 = 0$$

$$\frac{L_1}{L_2} = \left(\frac{D_1}{D_2}\right)^2 \quad \Rightarrow \quad \left(\frac{D_1}{D_2}\right) = \left(\frac{L_1}{L_2}\right)^{1/2} = 11.2$$

6. One last problem. From the H-R diagram, read off the approximate luminosity of (a) a red supergiant, (b) a red giant, and (c) a red dwarf.

Make sure that your reading is done at approximately the same temperature.

$$T = 4500\,\mathrm{K}, L_{\mathrm{sg}} = 10^5 L_\odot, L_{\mathrm{g}} = 10^2 L_\odot, L_{\mathrm{d}} = 0.1 L_\odot$$

a) If all three stars were at a distance of 10 parsecs, what would their apparent magnitudes be?

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{T}{T_\odot}\right)^4 \left(\frac{R}{R_\odot}\right)^2 \Rightarrow \left(\frac{R}{R_\odot}\right) = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T}{T_\odot}\right)^{-2}$$

$$\left(\frac{R_{sg}}{R_\odot}\right) = 562, \left(\frac{R_g}{R_\odot}\right) = 17.8, \left(\frac{R_d}{R_\odot}\right) = 0.56$$

b) If all three stars were at a distance of our Sun, how much brighter/fainter than the Sun would they be?

Note that apparent magnitude at 10pc is absolute magnitude!

$$M - M_0 = -\frac{5}{2} \log\left(\frac{L}{L_\odot}\right)$$

$$M_{sg} - M_\odot = -12.5, M_g - M_\odot = -5, M_d - M_\odot = +2.5$$